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Procedia Engineering 15 (2011) 5574 – 5578

Procedia Engineering

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Application Research on Electricity Demand Forecasting Based on Gaussian Quadrature Formula

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Abstract

This paper constructs the background value of grey GM (1, 1) model by using Gaussian quadrature formula, improves its accuracy and the data quality. It indicates that reconstruction of the background value is the key factor affecting prediction accuracy and applicability.

Keywords: GM (1, 1) model; Gauss quadrature formula; background value; forecasting

1. Introduction

Data prediction ability determines the quality of grid. As for electricity consumption, low prediction will cause power cut due to lack of allocated electricity, while high prediction will bring unnecessary generation cost and energy waste. Therefore, it is essential to predict the electricity consumption accurately. One of the commonest electricity consumption prediction models is grey GM (1, 1) model. Grey GM (1, 1) model can play a greater role in data forecast of electricity consumption.

In grey system theory, research on improvement and optimization of grey model is an important step solving its limitation and attracts wide attention of scholars. Depending on simulation experiment, literature [2] does quantitative research on application scope of GM (1, 1) model and the relation between developing coefficient and prediction accuracy; Literature [3] reconstructs the background value using Newton-Cores formula, Literature [4] reconstruct the background value with Lagrange formula and then improve the model; Literature [5] constructs the background value with exponential solution of first-order linear ordinary differential equations instead of consecutive neighbors mean in traditional model, of which the superiority is reducing the model error to some extent.

Because the concept of equidistance is introduced when constructing grey GM (1, 1) model, prerequisite making the model tenable is that the modeling sequence submits to requirement of equidistance. Nonetheless, there are many non-equidistant sequences in practical forecast problems. Thus, interpolation function $P(t)$ is constructed as the background value in the new state with Gauss integration, and is approximate to the background value $z^{(1)}(k+1)$ in the interval [k,k+1]. This approach has the feature of high accuracy, and can solve infinite integral problems. Although calculation is slightly complex when nodes distribution is irregular, result can be obtained by assistance of computer. On condition of satisfying convergence, this method is high in algebraic precision and has small relative error comparing with previous interpolation methods, this method. It increases model stability, and is of simple calculation and easy to programming implementation. Finally, the simulation example demonstrates the effectiveness of the proposed approach.

2. Modeling idea of conventional GM (1, 1) model

First, introduce the modeling mechanism of traditional GM (1, 1) model.

Let $X^{(0)}=\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\}$ be the original series. Make one-accumulation of it and get:

$$X^{(1)}=\{x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n)\}$$

where $X^{(1)}(k)=\sum_{i=1}^{k}x^{(0)}(i)\quad(k=1,2,\cdots,n)$, $X^{(1)}(k)$ is called one-accumulation series of $X^{(0)}(k)$ denoted as $1-AGO$.

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doi:10.1016/j.proeng.2011.08.1034

$x^{(1)}$ satisfies the following grey differential equation, of which the winterization form is

$$\frac{dx^{(1)}}{dt} + ax^{(1)} = b \tag{1}$$

where a,b are parameter to be identified. a is called developing coefficient, and b is called grey input.

In order to estimate a,b, discretely process Eq.(1) and get:

$$\Delta\left(x^{(1)}(k+1) + aX^{(1)}(k+1)\right) = b \quad k = 1,2,\cdots,n-1 \tag{2}$$

where $\Delta\left(x^{(1)}(k+1)\right)$ is inverse accumulated generated on the $(k+1)th$ moment from generating sequence $x^{(1)}$. That is:

$$\Delta\left(x^{(1)}(k+1)\right) = x^{(1)}(k+1) - x^{(1)}\left(k\right) = x^{(0)}\left(k+1\right) \tag{3}$$

In the grey prediction, $x^{(1)}(k+1)$ in Eq.(2) is the background value of $dx^{(1)}/dt$ on the $(k+1)th$ moment. Generally, calculate their means and get:

$$z^{(1)}(k+1) = \frac{1}{2}[x^{(1)}(k) + x^{(1)}(k+1)], (k = 1,2,\cdots,n-1) \tag{4}$$

Induce Eq.(3) Eq.(4) into the following equations and get:

$$\begin{cases} z^{(1)}(2) = a\left[-\frac{1}{2}\left(x^{(1)}(1) + x^{(1)}(2)\right)\right] + b \\ z^{(1)}(3) = a\left[-\frac{1}{2}\left(x^{(1)}(2) + x^{(1)}(3)\right)\right] + b \\ \vdots \\ z^{(1)}(n) = a\left[-\frac{1}{2}\left(x^{(1)}(n-1) + x^{(1)}(n)\right)\right] + b \end{cases} \tag{5}$$

$$B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix}, Y_n = [x^{(0)}(2), x^{(0)}(3), \cdots, x^{(0)}(n)]^T \quad, \boldsymbol{\alpha} = (a,b)^T \quad,$$

then Eq.□ can be simplified as the following linear model $Y = B\boldsymbol{\alpha}$. And by least square estimation approach, it is get:

$$\boldsymbol{\alpha} = \left(B^T B\right)^{-1} B^T Y \tag{6}$$

Induce the parameter estimated from Eq.(6) into Eq.(1), obtain the discrete solution:

$$\hat{x}^{(1)}(k+1) = (x^{(0)}(1) - \frac{b}{a}) \cdot e^{-ak} + \frac{b}{a} \tag{7}$$

Implement inverse accumulation on $\hat{x}^{(1)}(k+1)$ will get the prediction series:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = (1-e^{a})(x^{(0)}(1) - \frac{b}{a}) \cdot e^{-ak} \tag{8}$$

and $k = 1,2,\cdots,n$.

Through this modeling process, we find developing coefficient a and grey input b have a profound impact on the prediction accuracy of GM (1, 1).

3. The optimization and improvement of GM (1, 1)

The original mode uses consecutive neighbors mean to construct the background value

$$z^{(1)}(k+1) = \frac{1}{2}[x^{(1)}(k) + x^{(1)}(k+1)],$$

then exploit trapezoid area

$$S(k \cdot x^{(1)}(k) \cdot x^{(1)}(k+1) \cdot (k+1))$$

to replace the area that surrounded by curve $x^{(1)}(t)$

$$S(k \cdot \overbrace{x^{(1)}(k) \cdot x^{(1)}(k+1)}^{Arc} \cdot (k+1))$$

As shown in Figure 1.The method has some defects, with the index growing, data sequence changes intensify and the prediction deviation will be enlarged (ΔS), and affect the suitability of the model to some extent.

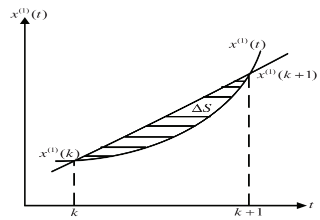

Fig. 1. Prediction deviation of GM (1, 1)

To overcome this deficiency, We use fivepoint gaussian quadrature to reconstruct the background value, in order to reduce the deviation. Firstly, we change the form of whiten differentiation equation.

Do the integral operation on both sides of system（1）in the interval [k,k+1].,we will obtain:

$$\int_k^{k+1} \frac{dx^{(1)}}{dt} dt + a\int_k^{k+1} x^{(1)} dt = b$$

That is

$$x^{(1)}(k+1) - x^{(1)}(k) + a\int_k^{k+1} x^{(1)} dt = b\,.$$

Namely,

$$x^{(1)}(k+1) + a\int_k^{k+1} x^{(1)} dt = b \tag{9}$$

From □ we know, the background value is

$$z^{(1)}(k+1) = \int_k^{k+1} x^{(1)} dt \tag{10}$$

Next, do some numerical treatment on background value, algorithm steps are as follows:

Step 1: Let $f(t) = x^{(1)}(t)$

then

$$\begin{aligned} \int_k^{k+1} x^{(1)}(t) dt &= \int_k^{k+1} f(t) dt = \frac{1}{2}\int_{-1}^{1} f\left(\frac{1}{2}u + k + \frac{1}{2}\right) du \\ &= \int_{-1}^{1} f(v) dv = A_0 f(v_0) + A_1 f(v_1) + A_2 f(v_2) + A_3 f(v_3) + A_4 f(v_4) \end{aligned} \tag{11}$$

Step 2: We know Gauss points are the zero points of Legendre polynomials, so let

$$P_5(v) = \frac{1}{8}\left(63v^5 - 70v^3 + 15v\right) = 0$$

will get

$$v_0 = -0.9061799, v_1 = -0.5384693, v_2 = 0, v_3 = 0.5384693, v_4 = 0.9061799$$

Step 3: In regard to the twice algebraic accuracy of Gauss-Legendre polynomial quadrature, the integrations $f(v) = 1, v, v^2, v^3, v^4$ are all accurately established. Therefore, imultaneous equations are:

$$\begin{cases} A_0 + A_1 + A_2 + A_3 + A_4 = \int_{-1}^{1} dv = 2 \\ -0.9061799A_0 - 0.5384693A_1 + 0A_2 + 0.5384693A_3 + 0.9061799A_4 = \int_{-1}^{1} v dv = 0 \\ (-0.9061799)^2 A_0 + (-0.5384693)^2 A_1 + 0^2 A_2 + 0.5384693^2 A_3 + 0.9061799^2 A_4 = \int_{-1}^{1} v^2 dv = \frac{2}{3} \\ (-0.9061799)^3 A_0 + (-0.5384693)^3 A_1 + 0^3 A_2 + 0.5384693^3 A_3 + 0.9061799^3 A_4 = \int_{-1}^{1} v^3 dv = 0 \\ (-0.9061799)^4 A_0 + (-0.5384693)^4 A_1 + 0^4 A_2 + 0.5384693^4 A_3 + 0.9061799^2 A_4 = \int_{-1}^{1} v^4 dv = \frac{2}{5} \end{cases}$$

The solutions are:

$v_0 = v_4 = 0.2369269, \quad v_1 = v_3 = 0.4786287, v_2 = 0.5688889$

So Eq.(11)

$$\begin{aligned} &0.2369269 f(-0.9061799) + 0.4786287 f(-0.5384693) \\ &+0.5688889 f(0) + 0.4786287 f(0.5384693) + 0.2369269 f(0.9061799) \\ &= 0.2369269 x^{(1)}\left(k + \frac{1}{2}(1 - 0.9061799)\right) + 0.4786287 x^{(1)}\left(k + \frac{1}{2}(1 - 0.5384693)\right) \\ &+0.5688889 x^{(1)}\left(k + \frac{1}{2}\right) + 0.4786287 x^{(1)}\left(k + \frac{1}{2}(1 + 0.5384693)\right) \\ &+0.2369269 x^{(1)}\left(k + \frac{1}{2}(1 + 0.9061799)\right) \end{aligned}$$

Step 4: the Optimized background value:

$$\begin{aligned} &z^{(1)}(k+1) = \int_{k}^{k+1} x^{(1)}(t)dt \approx \int_{k}^{k+1} S_k(t)dt \\ &= 0.2369269 x^{(1)}\left(k + \frac{1}{2}(1 - 0.9061799)\right) + 0.4786287 x^{(1)}\left(k + \frac{1}{2}(1 - 0.5384693)\right) \\ &+0.5688889 x^{(1)}\left(k + \frac{1}{2}\right) + 0.4786287 x^{(1)}\left(k + \frac{1}{2}(1 + 0.5384693)\right) \\ &+0.2369269 x^{(1)}\left(k + \frac{1}{2}(1 + 0.9061799)\right) \end{aligned} \tag{12}$$

Eq.□ is the new background value which we use five-point Gaussian quadrature method improved the GM (1,1) model.

4. Data Simulation and accuracy comparison

Now using fivepoint Gaussian Quadrature to optimize the background value and C # program to realize the forecast data. Table 1 shows the calculated results.

Table 1. The former 8 data to predict the next two data

Initial Value	Forecast Value	Err
2140773	2140773	0
1503389	1625290	8.1
2133738	2033693	4.7
2105210	2042131	3
2251740	2050603	8.9
2172895	2059111	5.2
2640204	2467654	6.5
2365897	2076233	12.
1975497	2084847	5.5
1889394	2093497	4.7

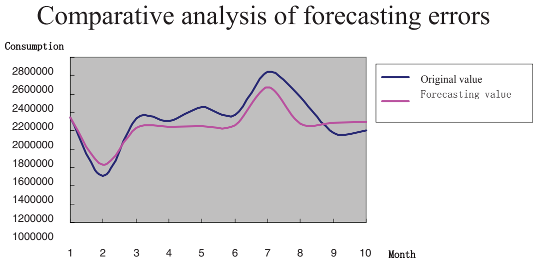

Fig. 2. Comparative analysis of forecasting errors

From the chart above,we can see the Gaussian quadrature method to forecast the predicted curve is similar to the original curve.

5. Conclusion

Simulation examples show a stronger applicability when using Gaussian Quadrature to optimize the the background value of the new GM (1,1) model. And will work more effectively in the smart grid and other practical applications.

Acknowledgements

This research was partially supported by the National Natural Science Foundation of China Grant No.71071045 and the Foundation of Hefei University of Technology Grant No.2010HGXJ0083.

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